Post-Quantum Isogeny-based Cryptography on ARM processors

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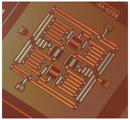
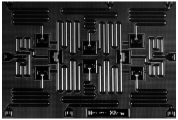
ARM Research Summit 2018

Why Quantum Computing? Why now?

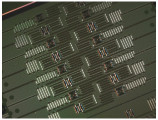
- The history of Integrated Circuits (IC)
  - 1958: First integrated circuit ($1cm^2$, 2 transistors)
  - 1971: Moore's Law is born (2,300 transistors)
  - 2014: IBM P8 Processor, 16 cores ($650mm^2$, $>$ 4.2 billion transistors)
- Quantum Computers[1]



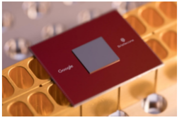
2015: 4-Qbit



2016: 8-Qbit

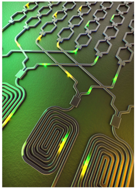


2017: 16-Qbit



2018: 72-Qbit

- Photon-based Quantum Computers are under construction!



[1] Pictures are taken from IBM Q Project

Quantum Computers

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[2] IBM's 50-bit Quantum Computer

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  - ECC: Elliptic Curve Discrete Logarithm Problem (ECDLP)

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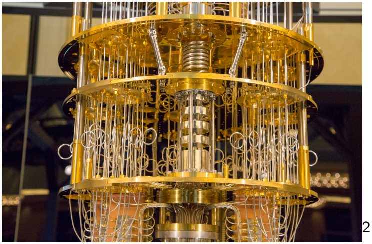
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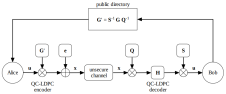

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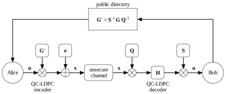

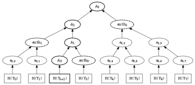

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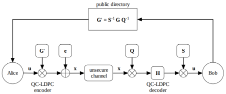

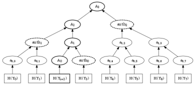

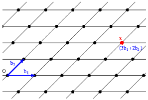

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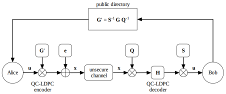

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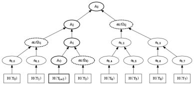

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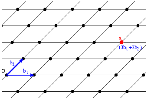

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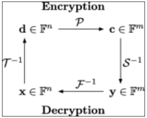

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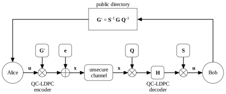

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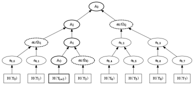

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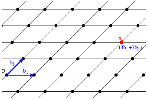

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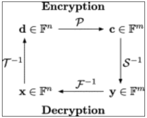
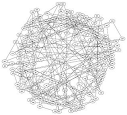
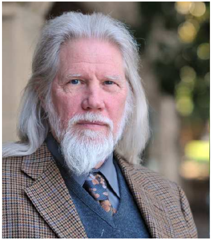


- Isogeny-Based: SIKE



Diffie-Hellman Key-Exchange Protocol

Secret Key
Public Key



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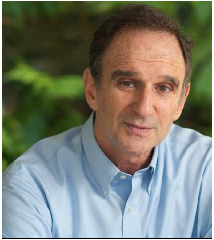[3]

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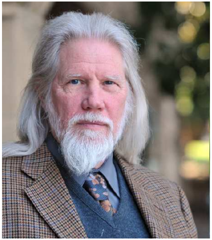

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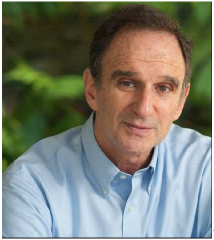
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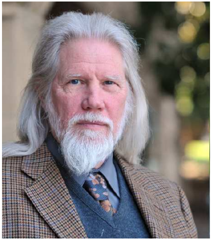

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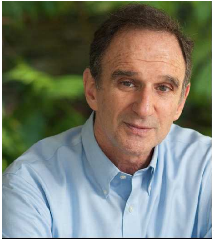
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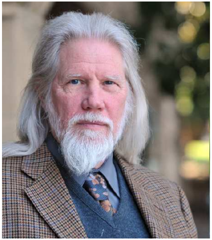

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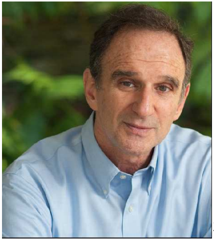
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Classical ECC vs. Post-Quantum Isogeny Cryptography

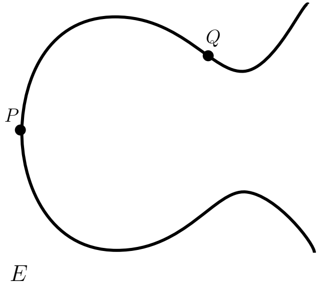
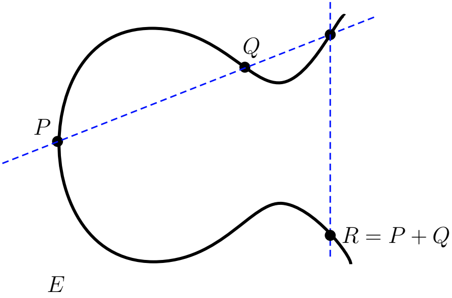


Figure: Classical Elliptic Curve Cryptography



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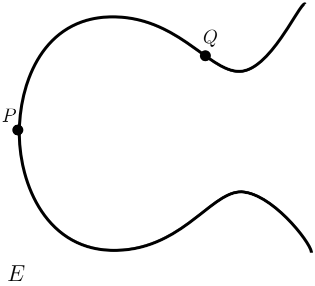

Figure: Post-Quantum Isogeny-based Cryptography

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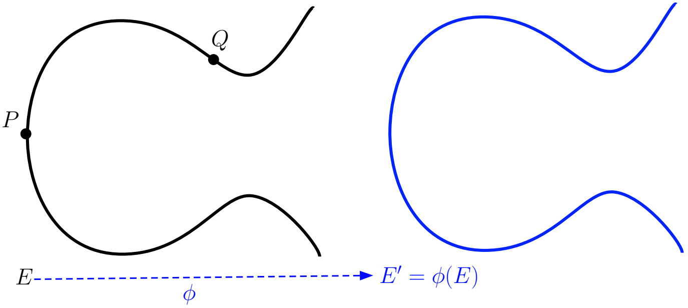

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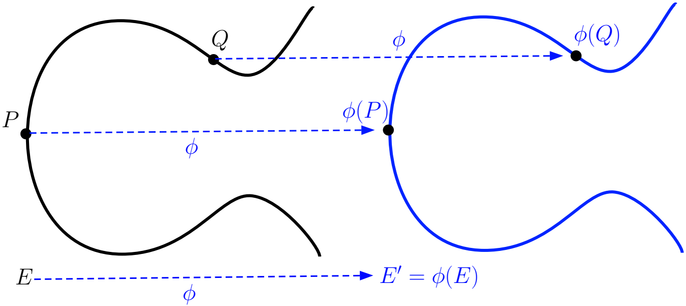

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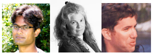

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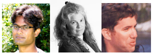

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- Supersingular Isogeny Key Encapsulation (SIKE) by Jao et al. submitted to NIST PQC Standardization 2017.

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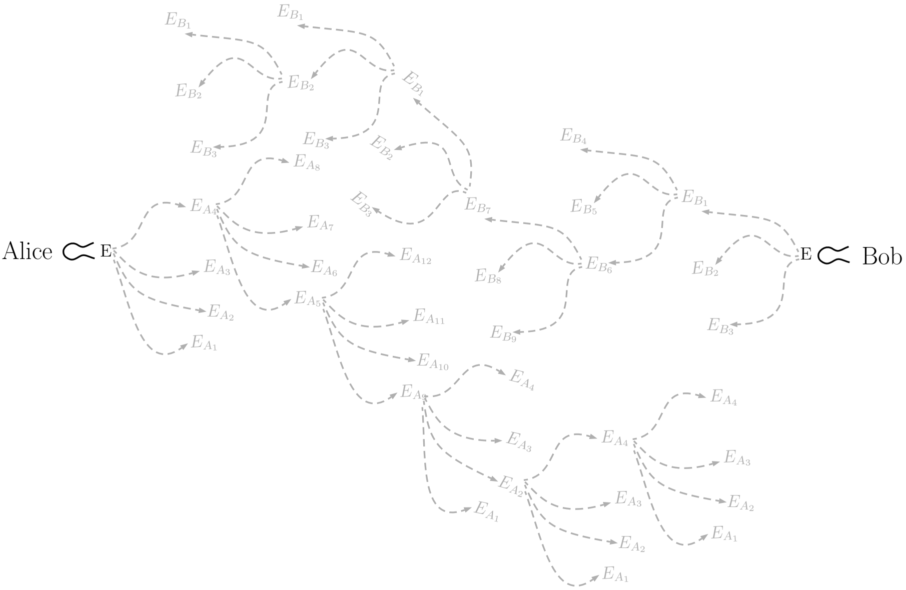
- Claw finding algorithm complexity for SIKE and SIDH:
  - $\mathcal{O}(p^{1/6}) \rightarrow$Quantum attacks
- The best known classical attack is based on meet in the middle
  - $\mathcal{O}(p^{1/4}) \rightarrow$Classical attacks

Alice and Bob Isogeny Walks from Different Degree Isogenies



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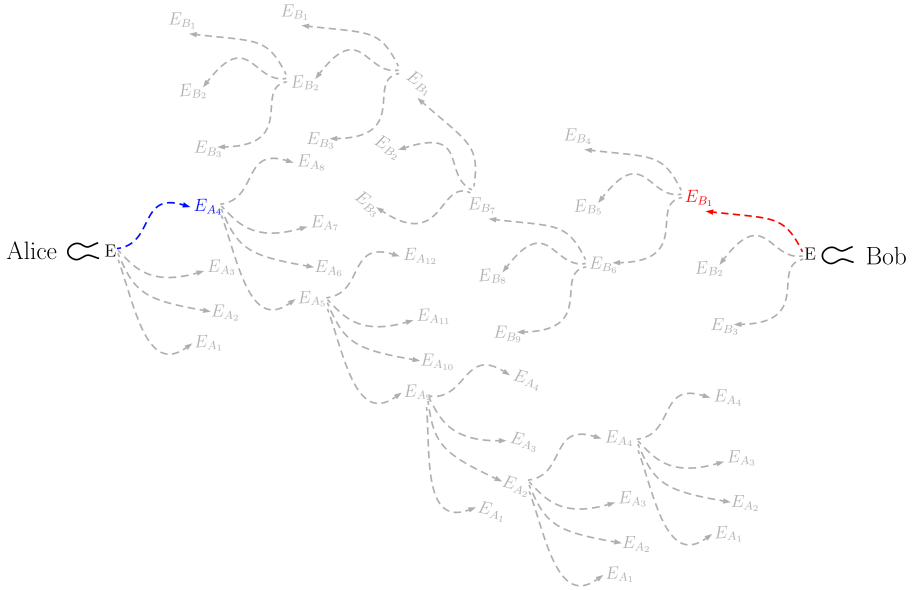

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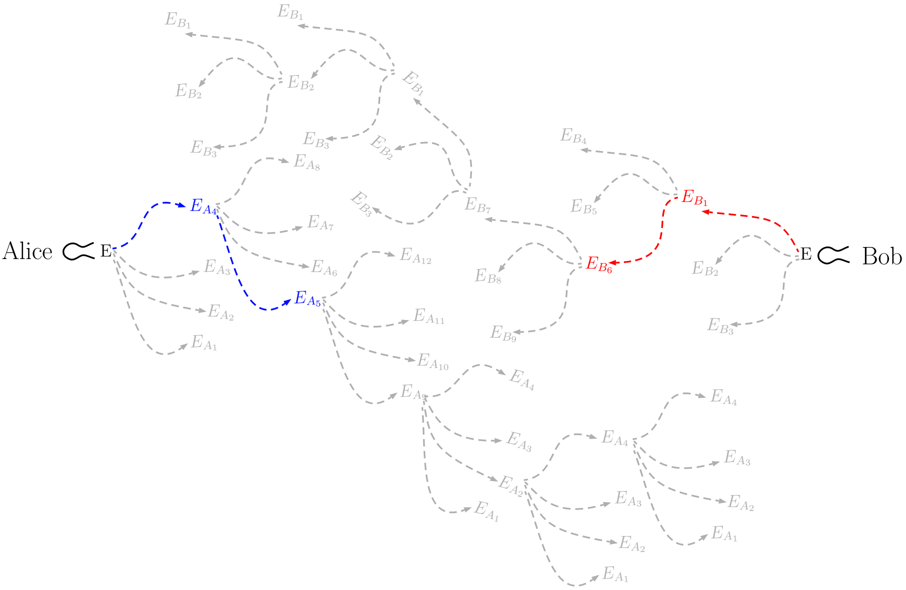

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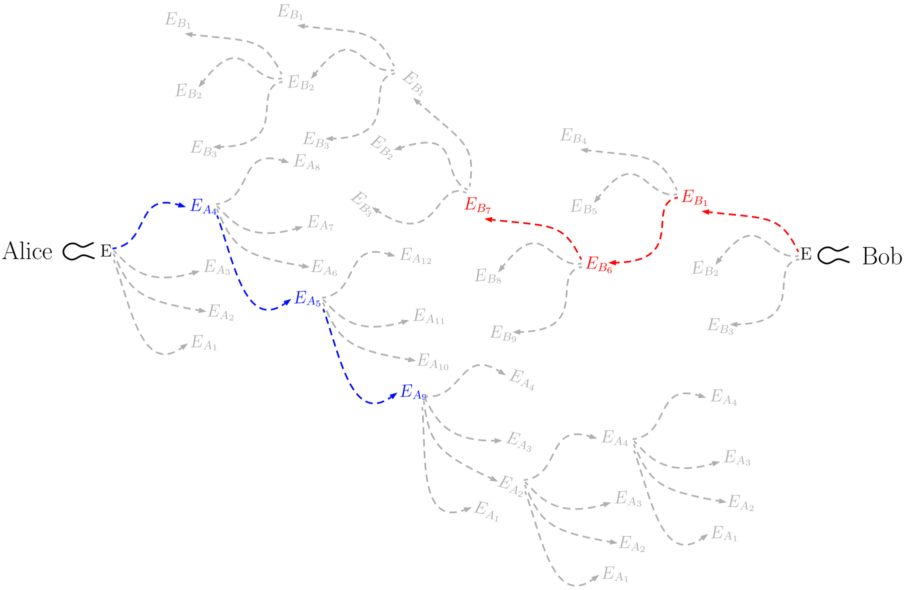

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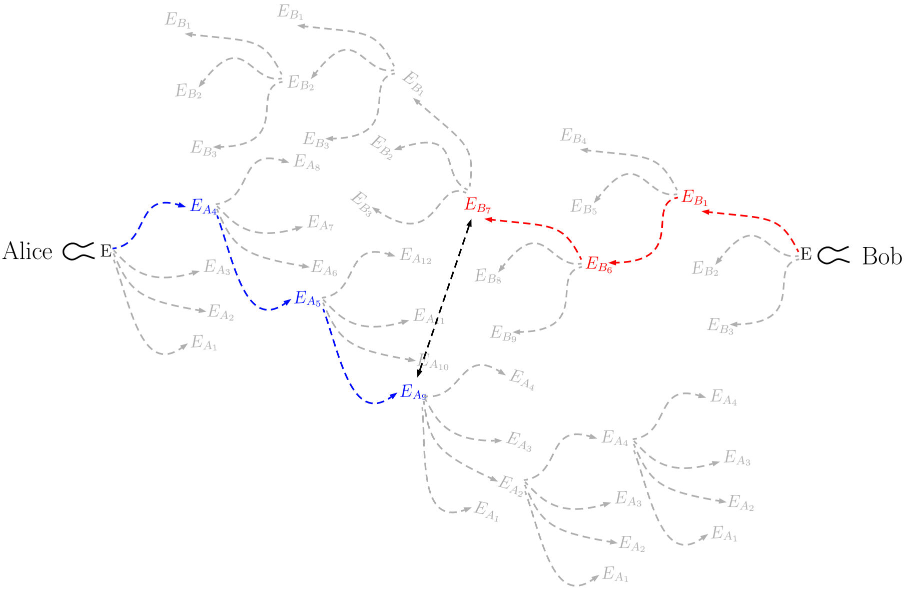

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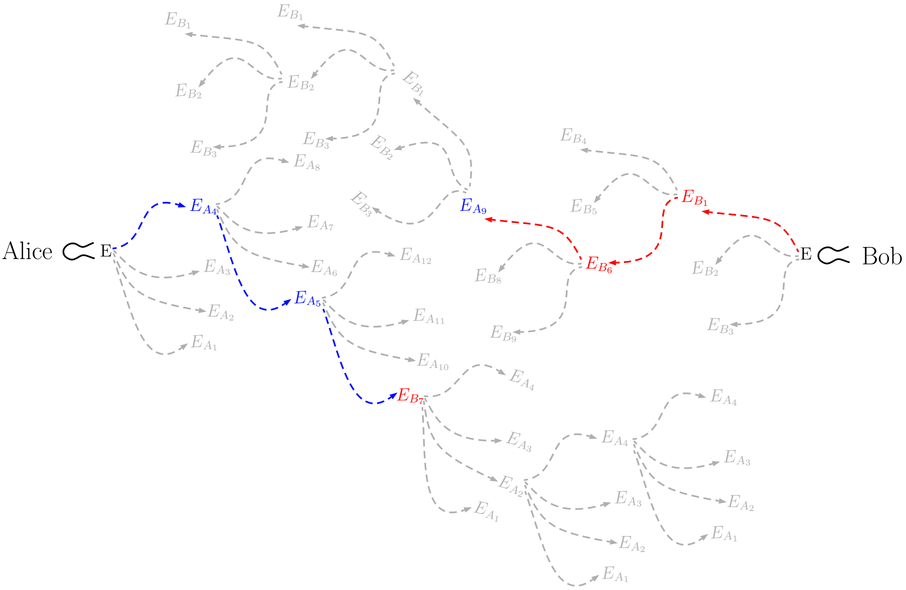

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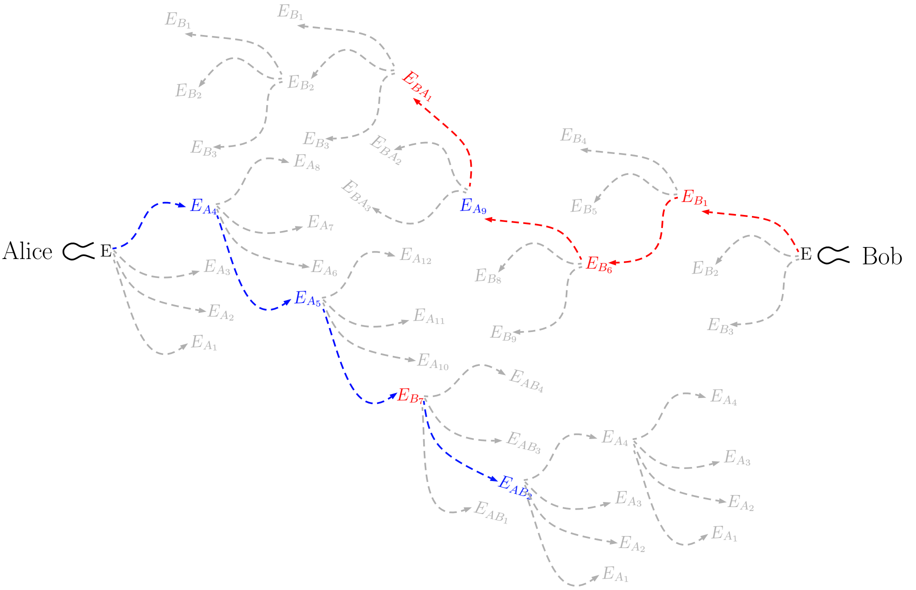

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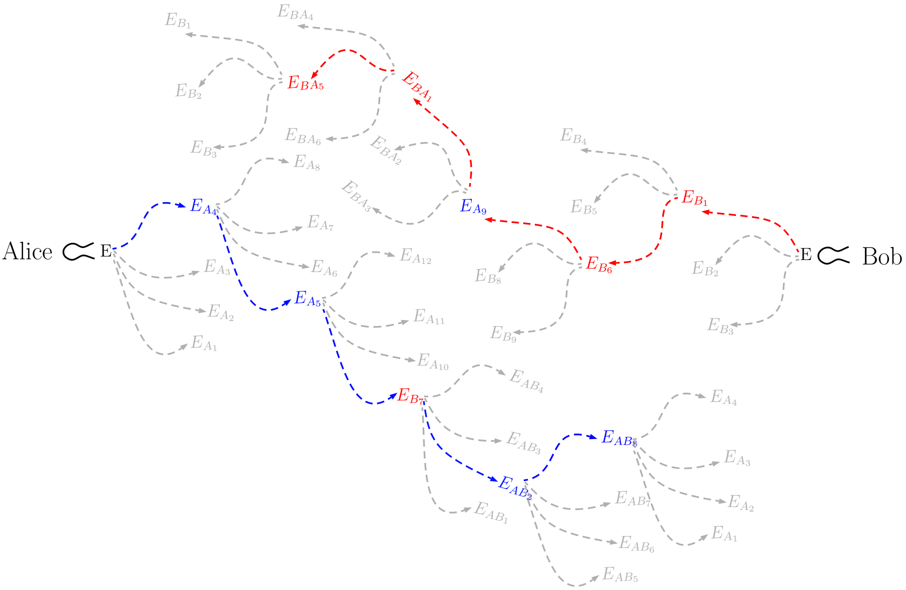

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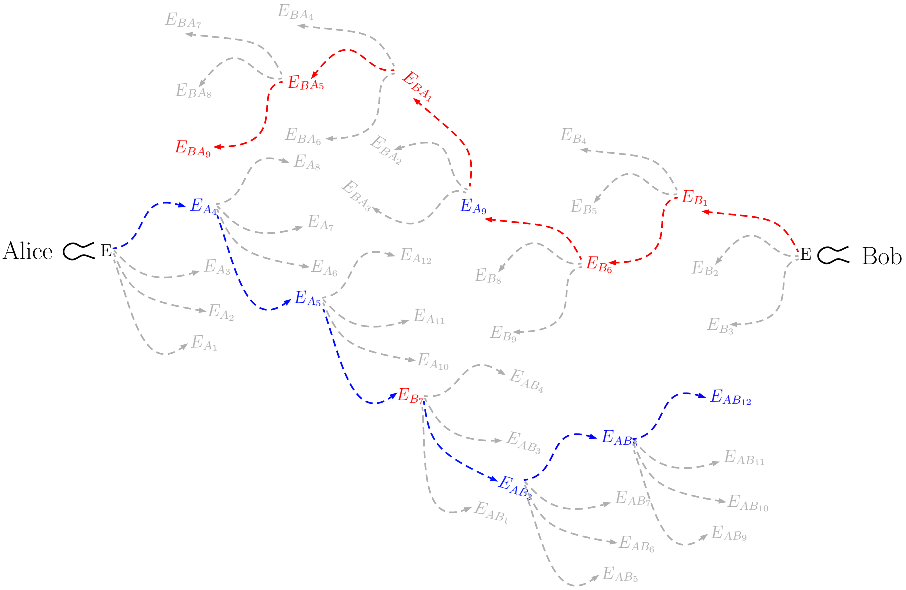

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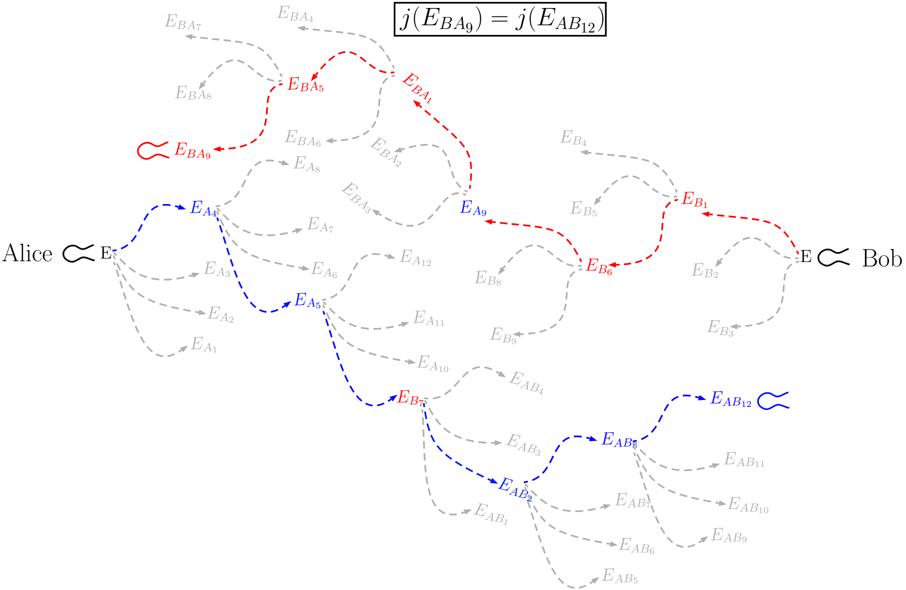

Public Parameters

E, p

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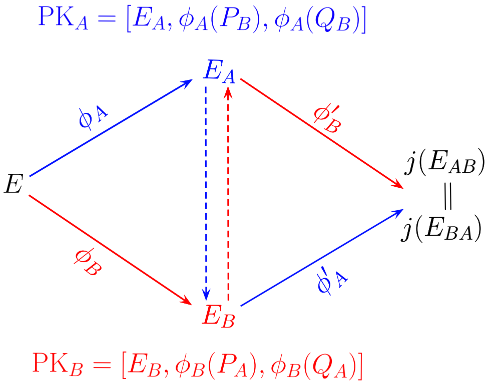
$P_B, Q_B \in E$



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  - Data-structure and implementation similar to ECC.
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Supersingular Isogeny-Based Cryptography Pros and Cons

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Small Key-Size Makes it Suitable for Embedded Devices

Table: Communication bandwidth of some NIST PQC candidate KEMs in terms of public-key, secret-key, and transmitted ciphertext during the key encapsulation process.

Candidate	Primitive	Size (Bytes)		
		Public key	Secret key	Ciphertext
NewHope1024	RLWE	1824	3680	2208
Saber	Mod-LWR	992	2304	1088
NTRU-HRSS17	LWE	1138	1418	1278
Kyber-768	LWE	1088	2400	1152
NTRU Prime	RLWE	1218	1600	1047
SIKEp751	SI	**564**	**644**	**596**

Isogeny-based Cryptography Implementation Perspective

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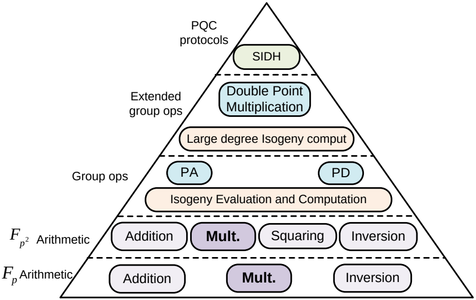

Different Families of Processors:

- ARMv7-M → 32-bit Low-Power (Performance is challenging)
- ARMv7-A → 32-bit High-Performance with NEON Instruction set
- ARMv8-A → 64-bit High-Performance with Adv. SIMD instruction set

[4]

[5]

Figure: ARMv7-A Cortex-A15 (Jetson TK1 Board) and ARMv8-A Cortex-A57 (Nexus smartphone)

[4] Taken from https://developer.nvidia.com

[5] Taken from https://www.huawei.com

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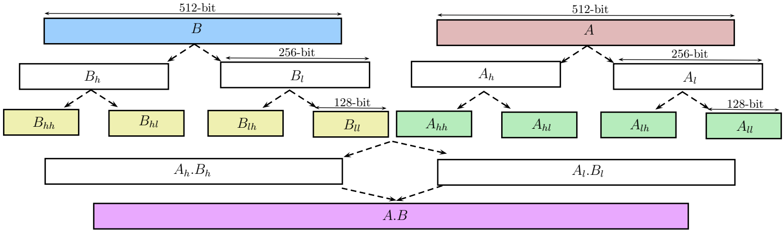

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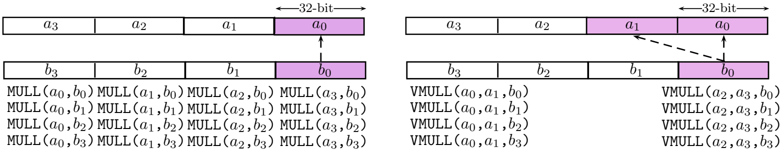

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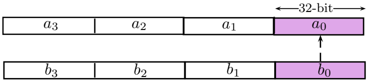

MULL(a_0,b_0)	MULL(a_1,b_0)	MULL(a_2,b_0)	MULL(a_3,b_0)
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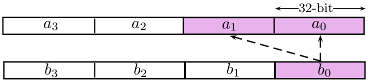

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VMULL(a_0,a_1,b_3)	VMULL(a_2,a_3,b_3)

- 16 $\times$ `MULL` instructions in A32 vs. 8 $\times$ `VMULL` in NEON Vector Instructions.

- SIDH performance evaluation on different families of ARM processors
- Different security levels

Work	Lang.	Device	Field size	PQ Security	Total Time (ms)
AFJ14	C	Cortex-A15	771	128	1,308
			1035	170	2,816
KJAJM16	ASM	Cortex-A15	1008	167	**982**
JAMJ17	ASM	Cortex-A57	751	125	**331**
			964	160	**652**
	C		751	125	1,846
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	ASM	Cortex-A72	751	125	**271**
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Performance Reports on Various Platforms

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- Reasonable performance, but we still need to improve these results.
- ARMv7-M platforms require further investigations due to the low working frequency (latest reported timings are in seconds).

- Quantum computers and their exceptional computational power will solve all the underlying problems that current PKC is constructed upon.
- We need to be prepared for this threat.
- NIST has already started the PQC standardization procedure.
- Different proposals have been submitted.
- SIKE is the only primitive which is constructed on the popular elliptic curves.
- SIKE offers the smallest key and ciphertext size among other candidates and it is suitable for embedded devices.

SIKE team

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Thank You!